Typesetting Musical Rhythms May Be Subtle

Jean-Michel Hufflen

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Introduction

Dividing by two

Time signatures

Using several time signatures

Irrational signatures

More developments

Conclusion
Memoizing music

Interesting for music already composed.
Memoizing music

Interesting for music already composed. Expresses the *structure* of a musical piece,
Memoizing music

Interesting for music already composed. Expresses the \textit{structure} of a musical piece, but what is displayed graphically should be preserved.
Memoizing music

Interesting for music already composed. Expresses the *structure* of a musical piece, but what is displayed graphically should be preserved. Example: multiple editions of Beethoven’s symphonies.
Designing new music

Designing, not composing, because that may be some new arrangement of an existing piece.
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Using some tools for checking that rhythms are correct, that is, successive bars are correctly filled in
Designing new music

*Designing*, not *composing*, because that may be some new arrangement of an existing piece. Using some tools for checking that rhythms are correct, that is, successive bars are correctly filled in (in addition to personal checking of what we hear).
Figures
Shortest duration’s figure

Theoretically $\frac{1}{64}$ sixty-fourth note.
Shortest duration’s figure

Theoretically \( \frac{1}{64} \) sixty-fourth note. But some (rare) examples using one-hundred-and-twenty-eighth notes.
Shortest duration’s figure

Theoretically ← sixty-fourth note. But some (rare) examples using one-hundred-and-twenty-eighth notes (Beethoven’s Emperor Concerto).
Western way

Dividing *long* musical rhythm figures originate from Western habits.
Western way

Dividing long musical rhythm figures originate from Western habits.
Some Asian or African music consider short durations and combine them.
Binary/ternary rhythms

Dotted note ↔ multiply its duration by 1.5 ('modern' interpretation):

\[
\begin{align*}
\dot{\circ} &= \circ + \cdot \\
\ddot{\circ} &= \circ + \cdot + \cdot \\
\dddot{\circ} &= \circ + \cdot + \cdot + \cdot \\
\end{align*}
\]
Binary/ternary rhythms

Dotted note \( \rightarrow \) multiply its duration by 1.5 (‘modern’ interpretation):

\[
\begin{align*}
\cdots &= \circ + \dot{\circ} \\
\cdots &= \circ + \dot{\circ} + \dot{\circ} \\
\ddots &= \circ + \dot{\circ} + \dot{\circ} + \dot{\circ} \\
\cdots &= \circ + \dot{\circ} + \dot{\circ} + \dot{\circ} + \dot{\circ} + \dot{\circ}
\end{align*}
\]
Binary/ternary rhythms

Dotted note $\leftarrow$ multiply its duration by 1.5 (‘modern’ interpretation):

$\circ \cdot = \circ + \uparrow$

$\circ \cdot \cdot = \circ + \uparrow + \uparrow$

$\vdots$

$\circ \cdot \cdot \cdot \cdot = \circ + \uparrow + \uparrow + \uparrow + \uparrow + \uparrow$

At the Middle Ages:

$Perfectus \leftarrow$ beat is divided by 3,

$imperfectus \leftarrow$ ................. 2.
Binary/ternary rhythms

Dotted note $\iff$ multiply its duration by 1.5 (‘modern’ interpretation):

\[
\begin{align*}
\cdot & = \cdot + \cdot \\
\cdot \cdot & = \cdot + \cdot + \cdot \\
\vdots & \quad \vdots \\
\cdot \cdot \cdot \cdot & = \cdot + \cdot + \cdot + \cdot + \cdot
\end{align*}
\]

At the Middle Ages:

\textit{Perfectus} $\iff$ beat is divided by 3,
\textit{imperfectus} $\iff$ ................. 2.

Nowadays: basic figure is dotted or not.
Irregular divisions

Example: *triplet*, instead of 2:

\[
\begin{array}{c}
\text{\includegraphics[width=0.3\textwidth]{example.png}}
\end{array}
\]
Irregular divisions

Example: *triplet*, instead of 2:

4-uplets, 5-uplets, 7-uplets, etc.
Unambiguous notation... for a musician.

Often numbers are dropped in case of repeated irregular groups...
Unambiguous notation . . .

. . . for a musician.
Often numbers are dropped in case of repeated irregular groups. . .
or in case of soli
Unambiguous notation. . .

. . . for a musician.
Often numbers are dropped in case of repeated irregular groups. . .
or in case of soli (Henri Tomasi’s Bassoon Concerto):
More

Durations *cumulated* in the complete figure.
Durations *cumulated* in the complete figure. Irregular groups may be *nested* (Arvo Pärt’s *Second Symphony*).
Rule for irregular groups

Put *more units* than normal division.
Rule for irregular groups

Put *more units* than normal division. Seven notes for a quarter note
Rule for irregular groups

Put *more units* than normal division.
Seven notes for a quarter note seven *eighteenth notes*.
Rule for irregular groups

Put *more units* than normal division.
Seven notes for a quarter note seven *eighteenth notes.*
Sometimes ← seven *thirty-second* notes, although it is incorrect.
Binary rhythm inside ternary context

Two notes for a quarter dotted note
Binary rhythm inside ternary context

Two notes for a quarter dotted note two \textit{quarter notes}. 
Binary rhythm inside ternary context

Two notes for a quarter dotted note two *quarter notes*. Often removing a dot may be observed.
Two notes for a quarter dotted note two *quarter notes*. Often removing a dot may be observed. Remark 2-, . . . , 4-, . . . not really needed $\implies$ dotted notes.
Complete notation

... for ...

Example of a 7-uplet.
Time signature

Numerator ← number of beats
Denominator ← figure for a beat
Time signature

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Denominator ← figure for a beat

Figure for a beat ← 1 for a whole note, 2 for a half note, etc. power of 2.
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Time signatures appeared in the 16th century, bar lines only in the 17th one.
Provided for beat 2, 3 or 4, and binary rhythms.
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Numerator ← number of beats
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*Time signatures* appeared in the 16th century, *bar lines* only in the 17th one.

Provided for beat 2, 3 or 4, and binary rhythms.

Ternary rhythms ← if the numerator is 6, 9 or 12, divide it by 3 and consider the third of the denominator figure.
Time signature

\[ \text{Numerator} \leftrightarrow \text{number of beats} \]
\[ \text{Denominator} \leftrightarrow \text{figure for a beat} \]

Figure for a beat \( \leftrightarrow \) 1 for a whole note, 2 for a half note, etc. power of 2.

Time signatures appeared in the 16th century, bar lines only in the 17th one.
Provided for beat 2, 3 or 4, and binary rhythms.
Ternary rhythms \( \leftrightarrow \) if the numerator is 6, 9 or 12, divide it by 3 and consider the third of the denominator figure.
Example: 6/8
Time signature

Numerator $\leftarrow$ number of beats
Denominator $\leftarrow$ figure for a beat

Figure for a beat $\leftarrow$ 1 for a whole note, 2 for a half note, etc. power of 2.

Time signatures appeared in the 16th century, bar lines only in the 17th one.
Provided for beat 2, 3 or 4, and binary rhythms.
Ternary rhythms $\leftarrow$ if the numerator is 6, 9 or 12, divide it by 3 and consider the third of the denominator figure.
Example: $6/8 \Longrightarrow 2\downarrow$. 
Abbreviations

Baroque music:

\[ 2 \Rightarrow 2/2 \quad 4 \Rightarrow 4/4 \]
Abbreviations

Baroque music:

\[ 2 \mapsto 2/2 \quad 4 \mapsto 4/4 \]

\(C\) common time (imperfectus) \(4/4\)

\(\|\) cut time (alla breve) \(2/2\)
Better notation

3/♩  2/♩
Better notation

\[\frac{3}{\dot{\text{m}}} \quad \frac{2}{\dot{\text{m}}}\]

Coined by French composer Jean-Philippe Rameau.
Better notation

\[3/\ldots\quad 2/\ldots\]

Coined by French composer Jean-Philippe Rameau. Used by Carl Orff.
The ‘classical’ notation is ambiguous

\[ \frac{6}{4} \rightarrow \text{beat } \frac{6}{\dot{}} \text{ or } \frac{2}{\dot{}}? \]
The ‘classical’ notation is ambiguous

6/4 \rightarrow \text{beat 6\textsuperscript{\Large\textbullet} or 2\textsuperscript{\Large\textbullet}?}

Classically, there is no beat-6 signature, but what about 18/8?
Other beats

5, 6, 7 and more... using *dotted* bar lines to emphasise division.
Other beats

5, 6, 7 and more... using *dotted* bar lines to emphasise division.

(Arvo Pärt’s *First Symphony*):

![Musical notation example](image-url)
Other beats

5, 6, 7 and more... using *dotted* bar lines to emphasise division. (Arvo Pärt’s *First Symphony*):

9, 12...
Using sums

Expressing (asymmetrical) division, e.g.:

\[
\frac{3 + 3 + 2}{8} \text{ instead of } \frac{8}{8}
\]
Added value

Adding a sub-value at the extreme part, e.g.:

\[
3 + \frac{1}{2} = 4
\]

(Edgar Varèse’s *Intégrales*)
Overlapping accents

\[
\begin{align*}
2/4 & \quad \times \quad \times \quad \times \quad \times \quad \times \quad \ldots \\
3/4 & \quad \times \quad \times \quad \times \quad \times \quad \times \quad \times \quad \ldots
\end{align*}
\]
### Equivalence between bars

**Same beat:**

\[
\begin{align*}
3/\text{ } & \quad \quad \text{ } \quad \quad \text{ } \quad \quad \text{ } \\
\text{ } & \quad \quad \text{ } \quad \quad \text{ } \quad \quad \text{ } \\
\end{align*}
\]

\[
\begin{align*}
\quad \quad \text{ } \quad \quad \quad \quad \text{ } \quad \quad \text{ } \\
\end{align*}
\]


Equivalence between bars

Same beat:

\[
\begin{array}{c|cccccc}
3/\d & \sp & \sp & \sp & \sp & \sp & \sp & \ldots \\
3/\d. & \sp & \sp & \sp & \sp & \sp & \sp & \ldots
\end{array}
\]

or not:

\[
\begin{array}{c|cccccc}
3/\d & \sp & \sp & \sp & \sp & \sp & \sp & \ldots \\
2/\d. & \sp & \sp & \sp & \sp & \sp & \sp & \ldots
\end{array}
\]
Irrational time signatures

Coined by some modern British composers: Brian Ferneyhough, Thomas Adès, ... Avoids the notation of added values.
Example

\[ \frac{5}{2} \times \frac{3}{2} = \frac{15}{4} \]
Example

Before: \[\frac{3}{4} \] instead of \[\frac{2}{4} \times \frac{3}{2} = \frac{3}{4} \].

is equivalent to:
Example

is equivalent to:

Before: $3 \ \underline{\text{¶}}$ instead of $2 \ \underline{\text{¶}}$. 
Example

is equivalent to:

Before: 3 \( \updownarrow \) instead of 2 \( \updownarrow \).

\[
\frac{4}{2} \times \frac{2}{3} = \frac{4}{3}
\]
Example (continued)
More accurate

Thomas Adès’s *Catch*:

Number of elements of *triplets* $\leftarrow 12$ for a $\circ$
Mathematically

\[
\frac{1 + \frac{2}{3}}{4} = \frac{5}{3} \times \frac{1}{4}
\]
Mathematically

\[ \frac{1 + \frac{2}{3}}{4} = \frac{5}{3} \times \frac{1}{4} \]

\[ = \frac{5}{12} \]
Free and random rhythms

Luciano Berio’s *Folk Songs*:
My advice for conceptors of music software

Clearly separate structure and layout.
My advice for conceptors of music software

Clearly separate structure and layout.
Encourage experiment!