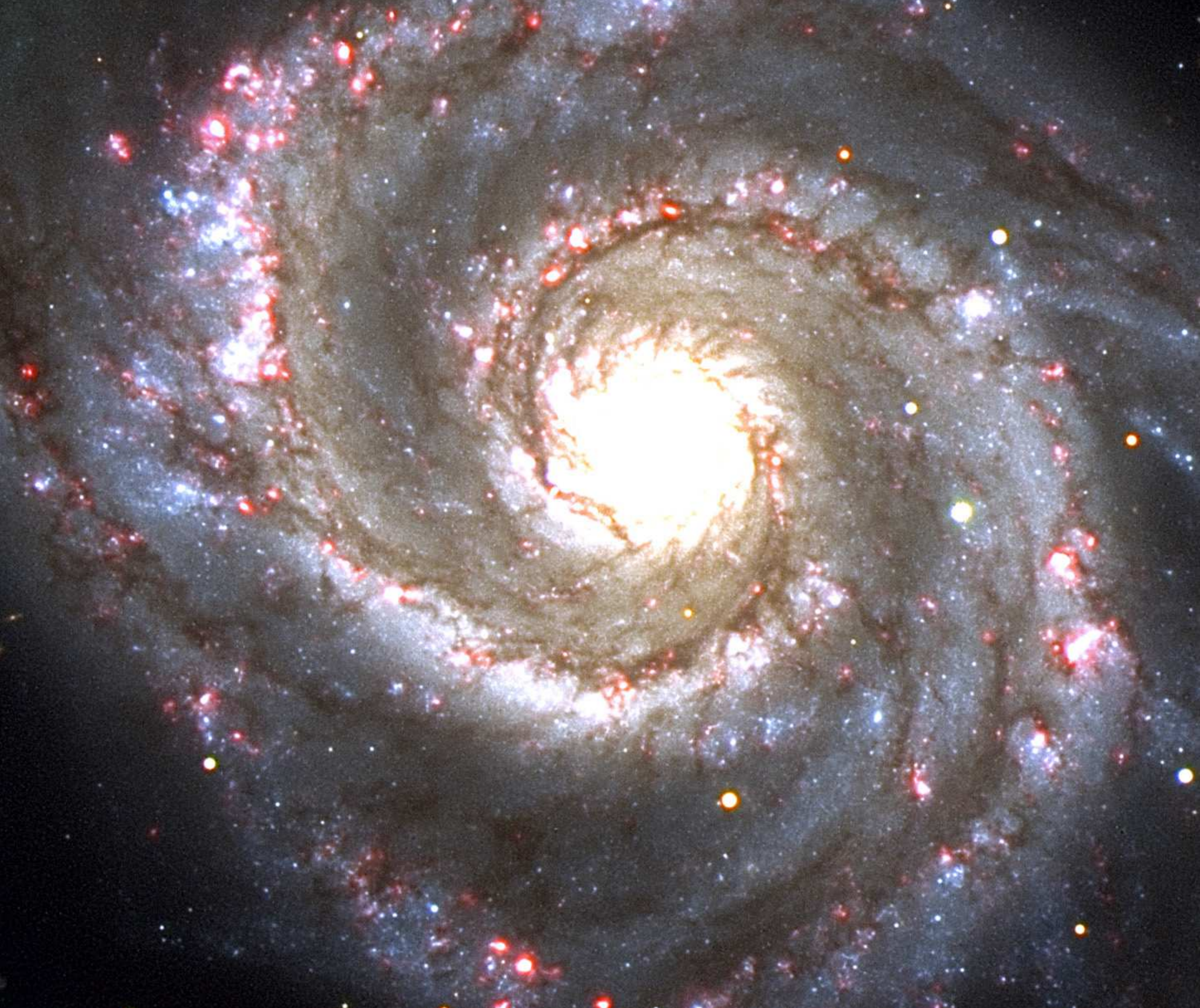

How to compute a winding angle and an area?

Bachotek 29 IV – 3 V 2011

Bogusław Jackowski

What is a winding angle? And a winding number?

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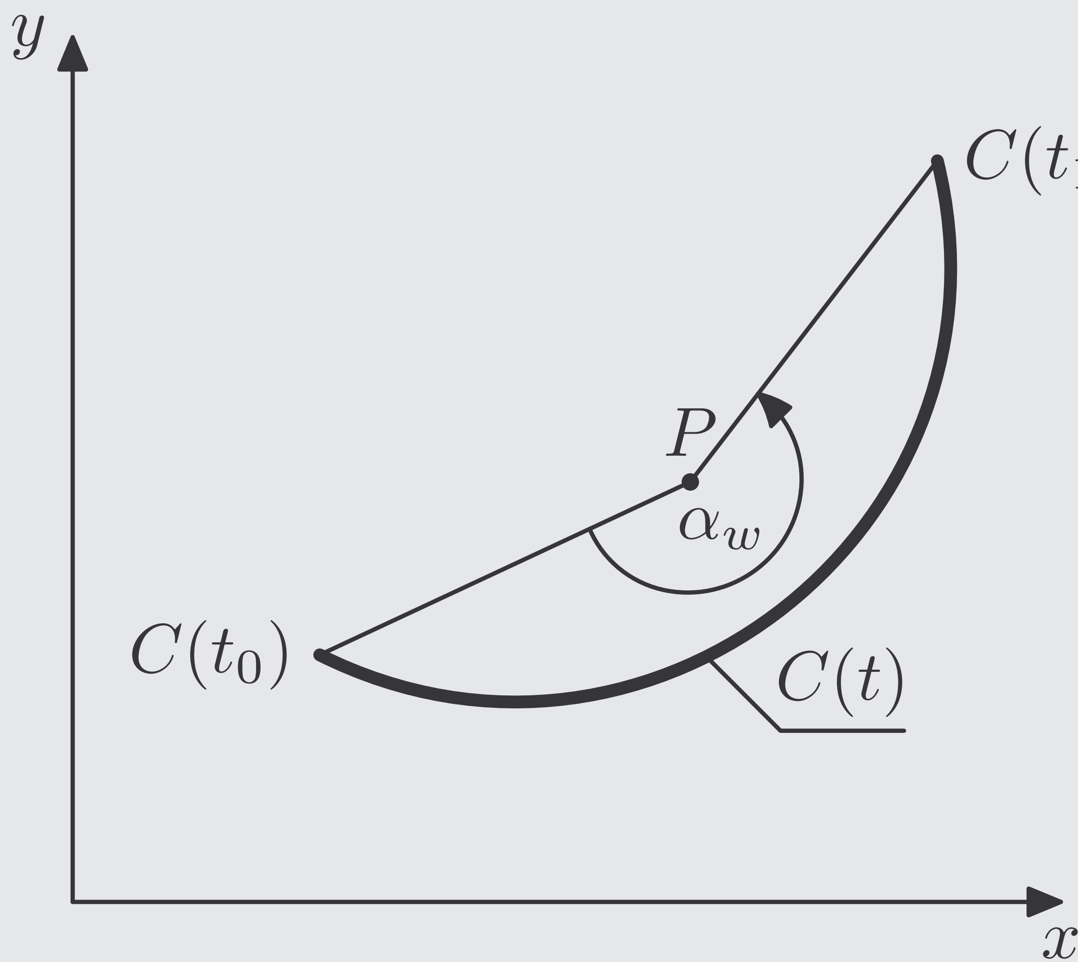

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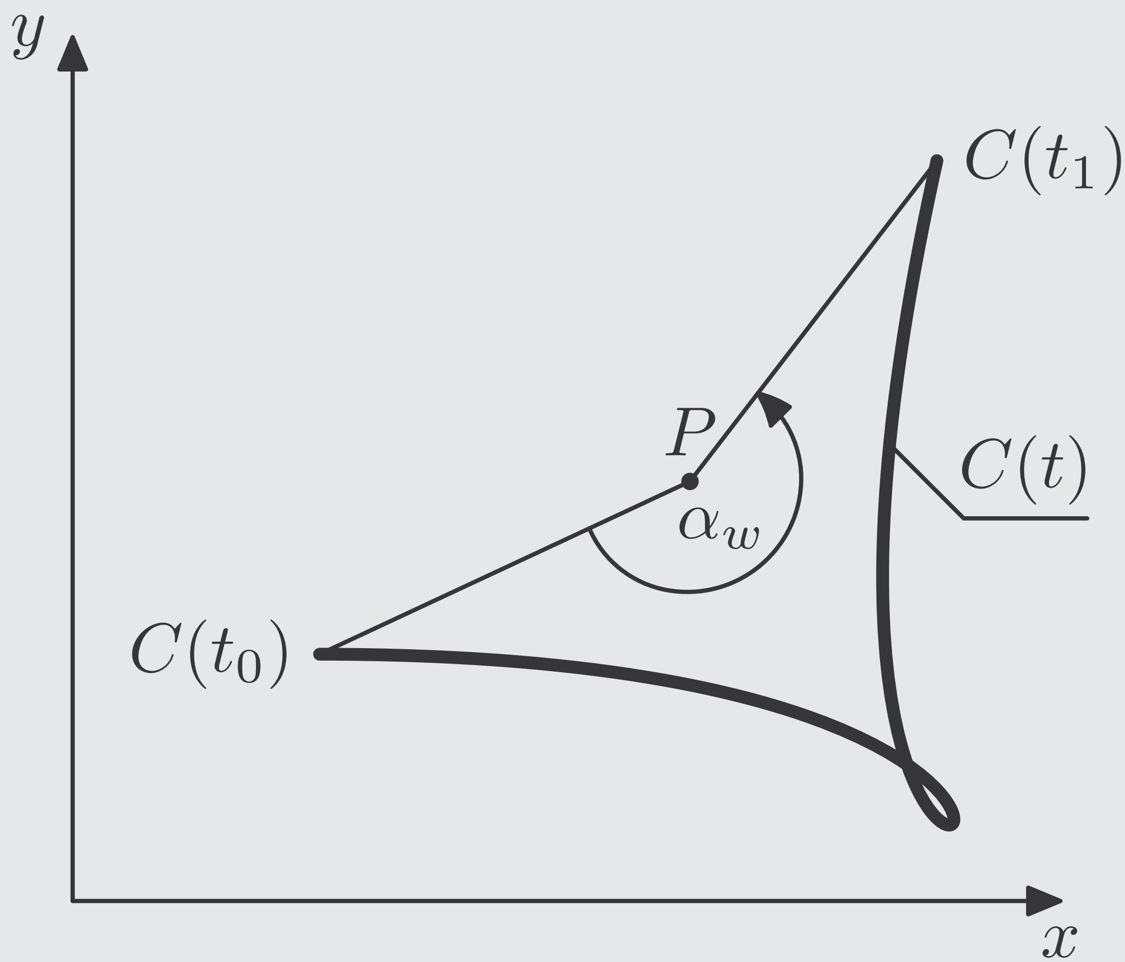

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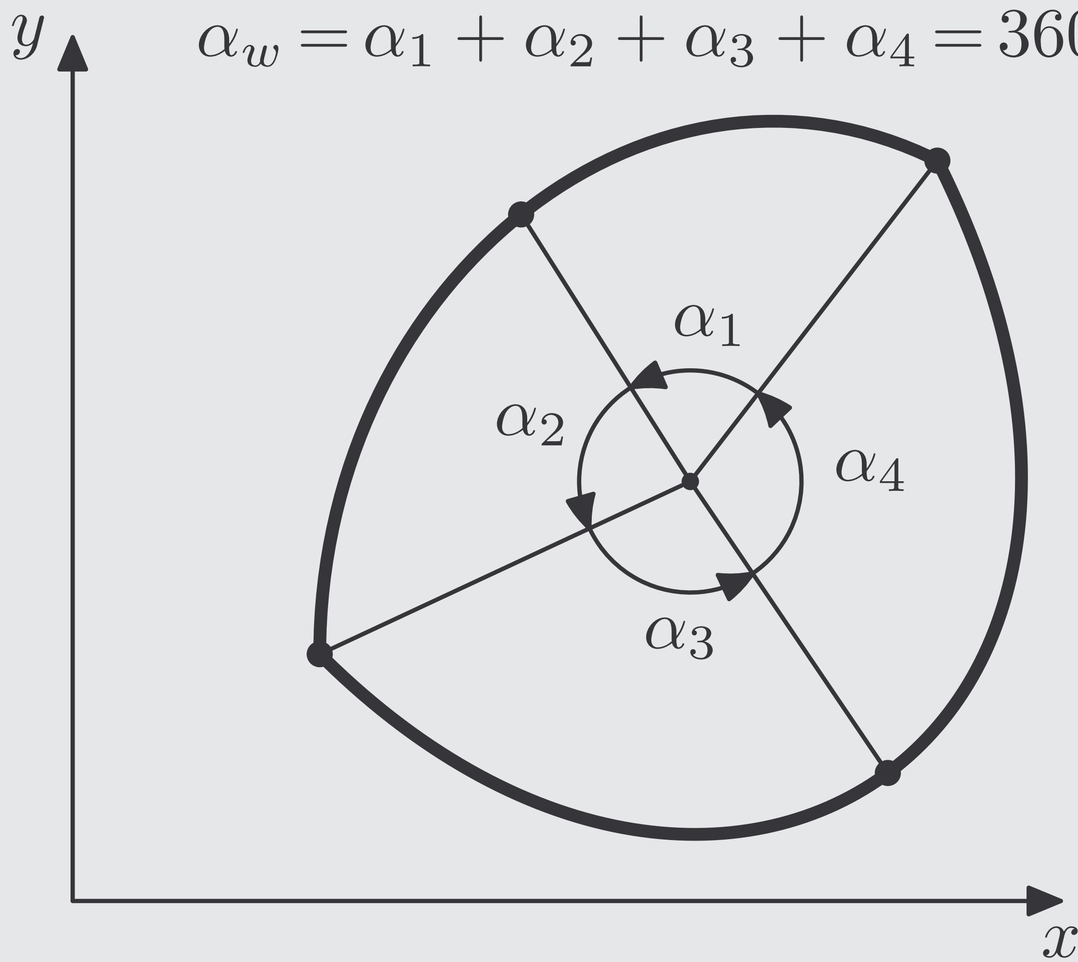

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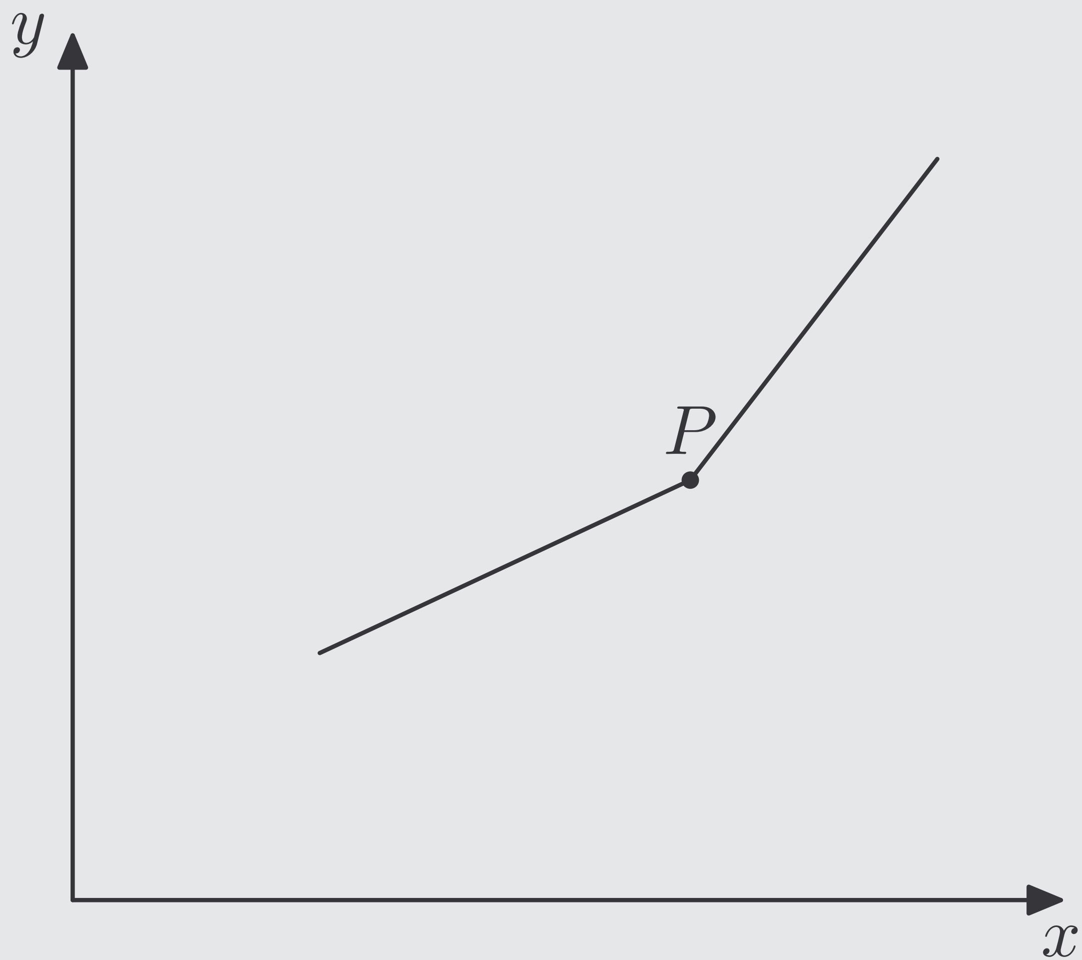

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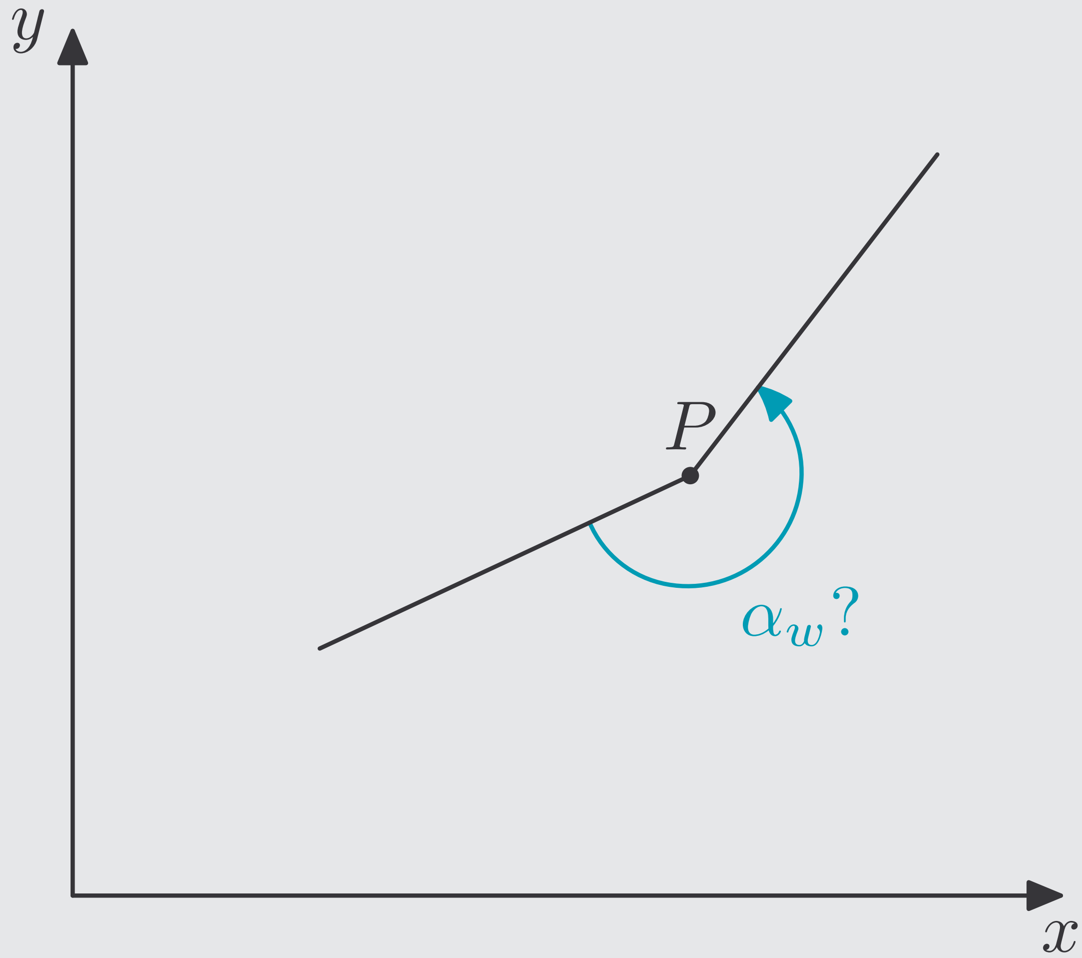

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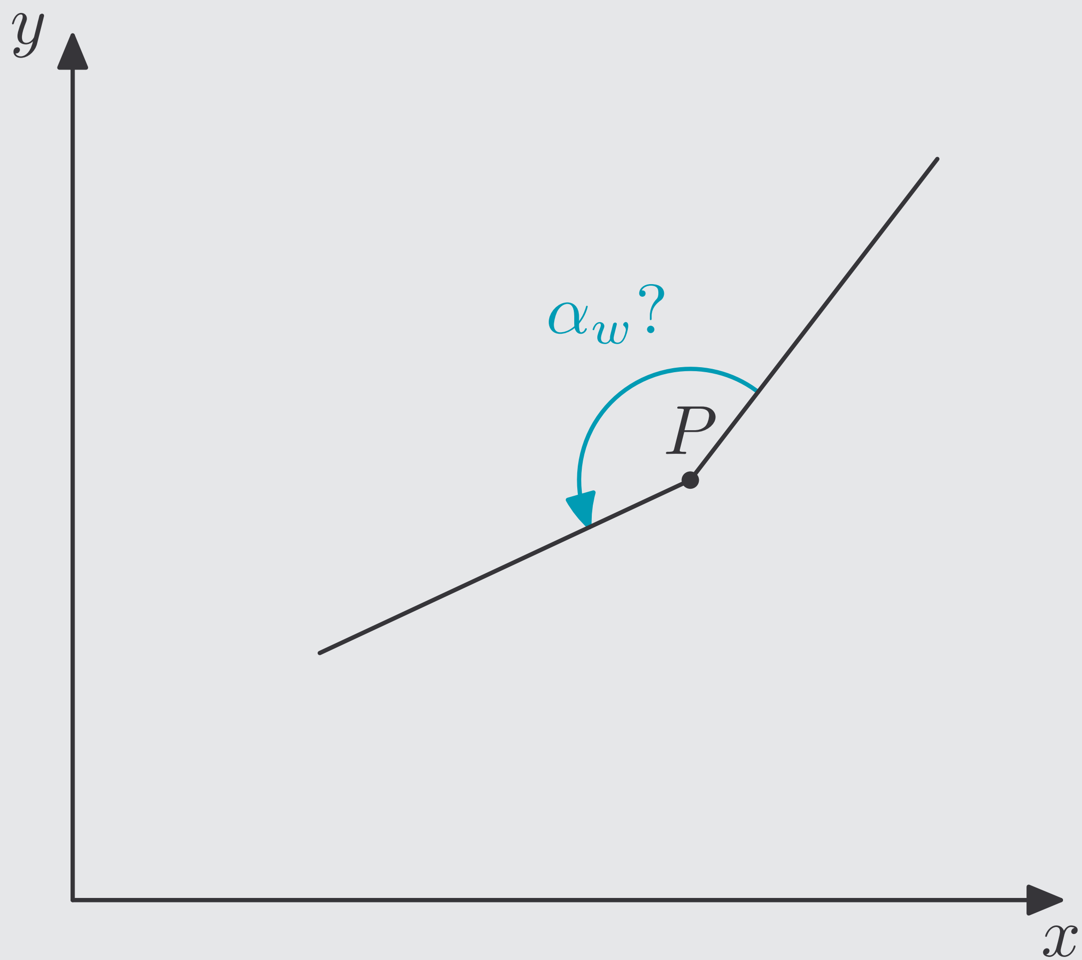

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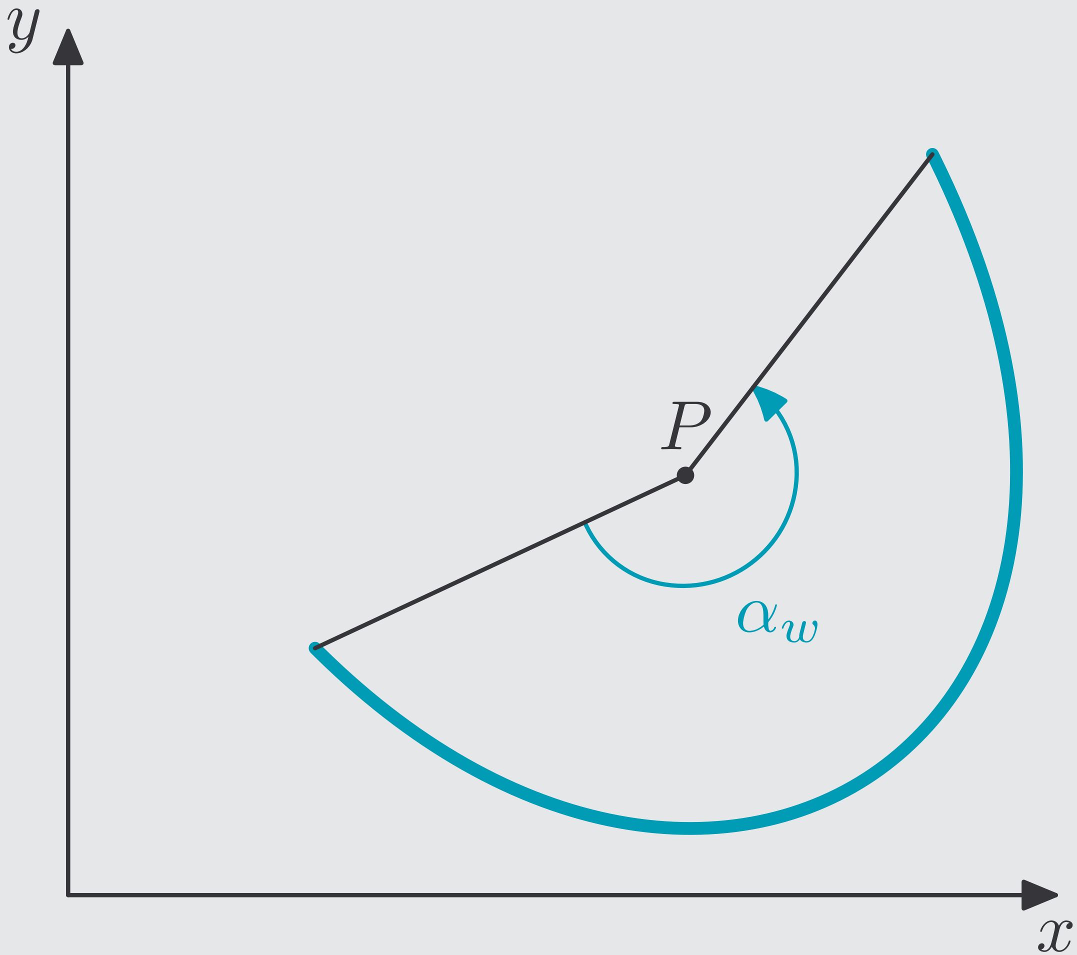

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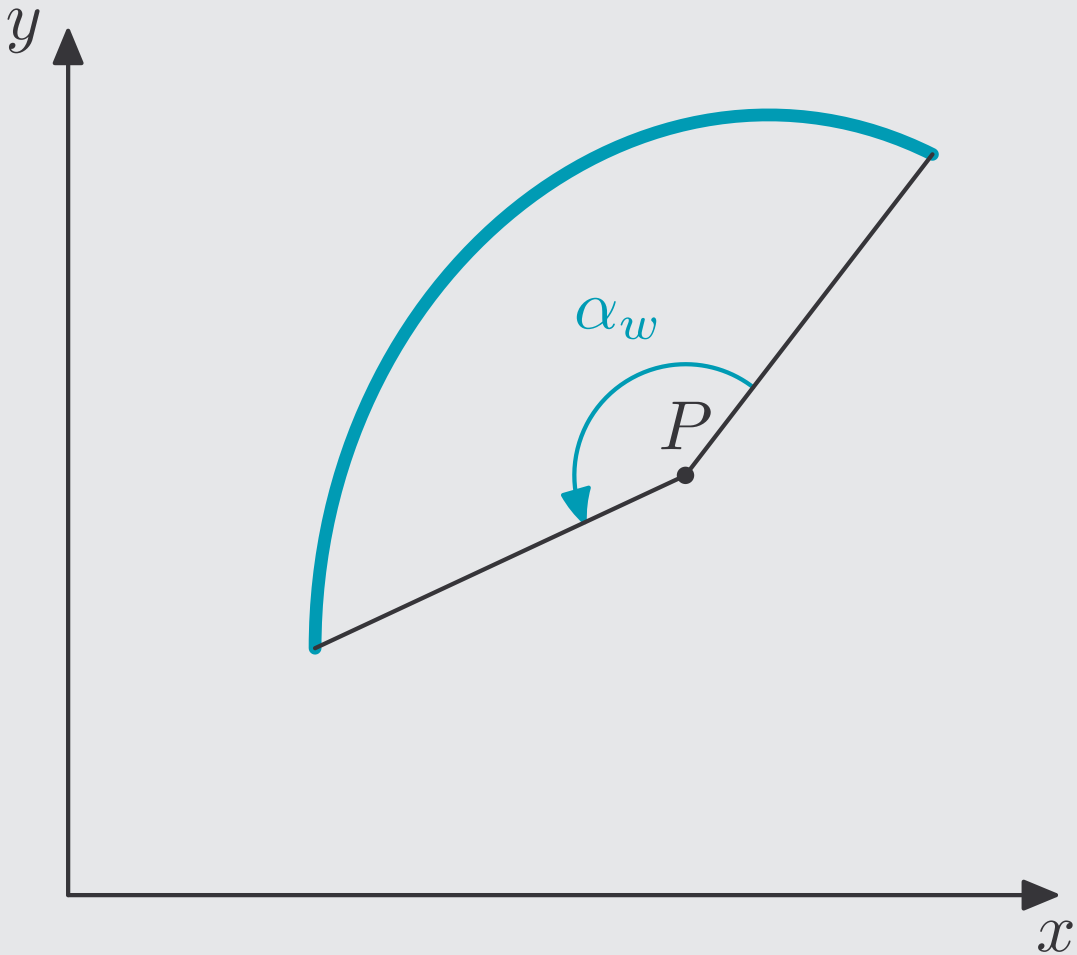

Crucial point

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The proper angle cannot be calculated without the analysis of the shape of the Bézier segment.

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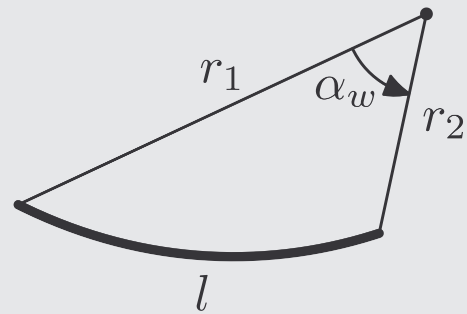
Of course, we can refer to some global properties of the arc. As Larry Siebenmann pointed out, if the length of the arc is less than the length of the longer radius, we can safely assume that the angle in question is acute. Therefore, it can be determined unequivocally.

$$l < \max\{r_1, r_2\} \Rightarrow \alpha_w < 90^\circ$$

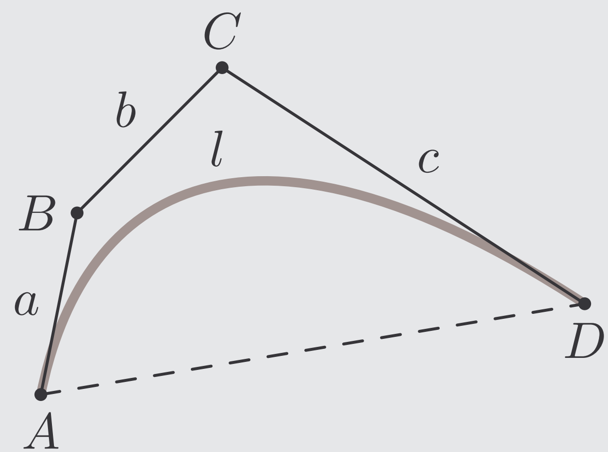


Crucial point

The proper angle cannot be calculated without the analysis of the shape of the Bézier segment.

Actually, calculating the length of the arc is a rather complex procedure and, moreover, unnecessary – the total length of the broken line joining the control nodes can be used as an adequate approximation.

$$l \leq a + b + c \stackrel{\text{def}}{=} \text{“mock length”}$$



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Actually, calculating the length of the arc is a rather complex procedure and, moreover, unnecessary – the total length of the broken line joining the control nodes can be used as an adequate approximation.

And that's all – the idea of the algorithm exploits this observation: the Beziér segment is bisected until the approximated ("mock") length of the arc is sufficently small.

Conclusion

Besides METAPOST operation *turningnumber* it would be nice to have a built-in operation *windingangle* or, equivalently, *windingnumber* as a useful tool for checking path properties.

An example: embedding

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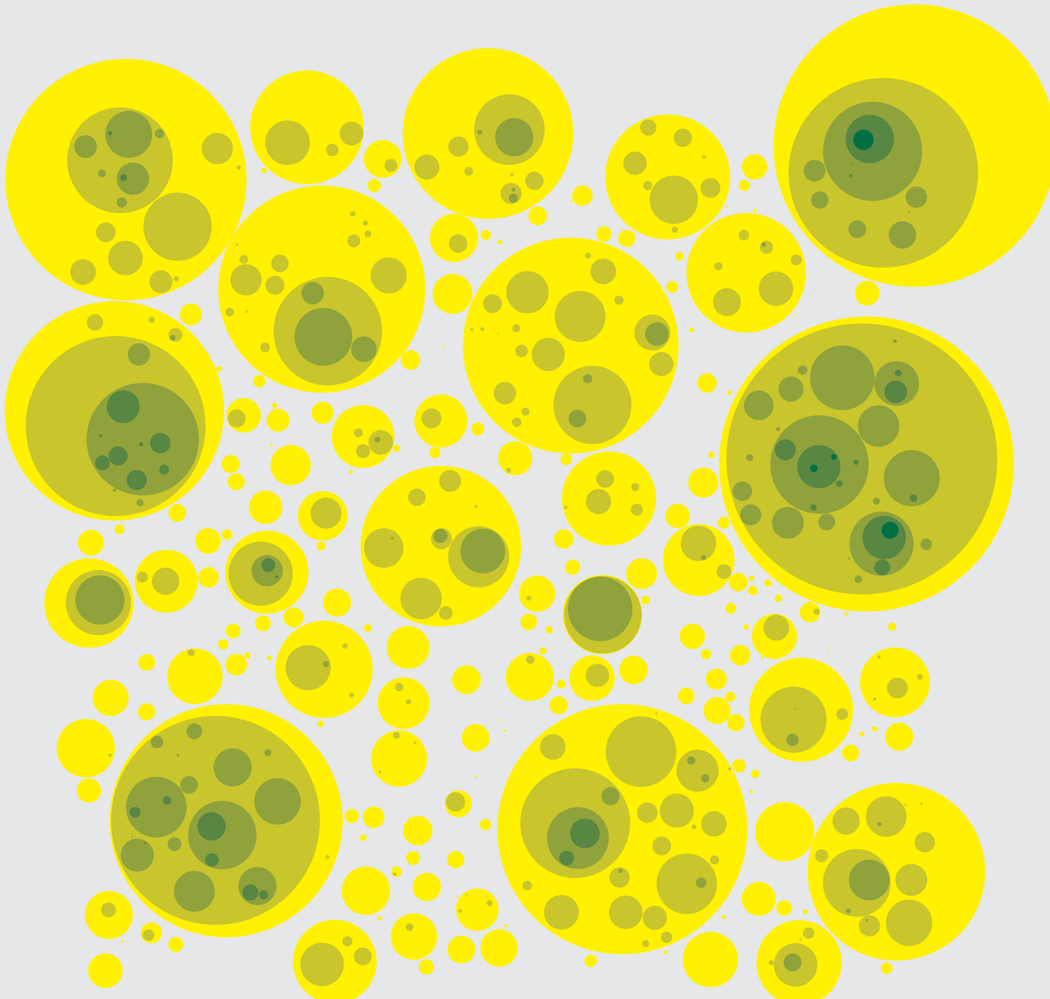

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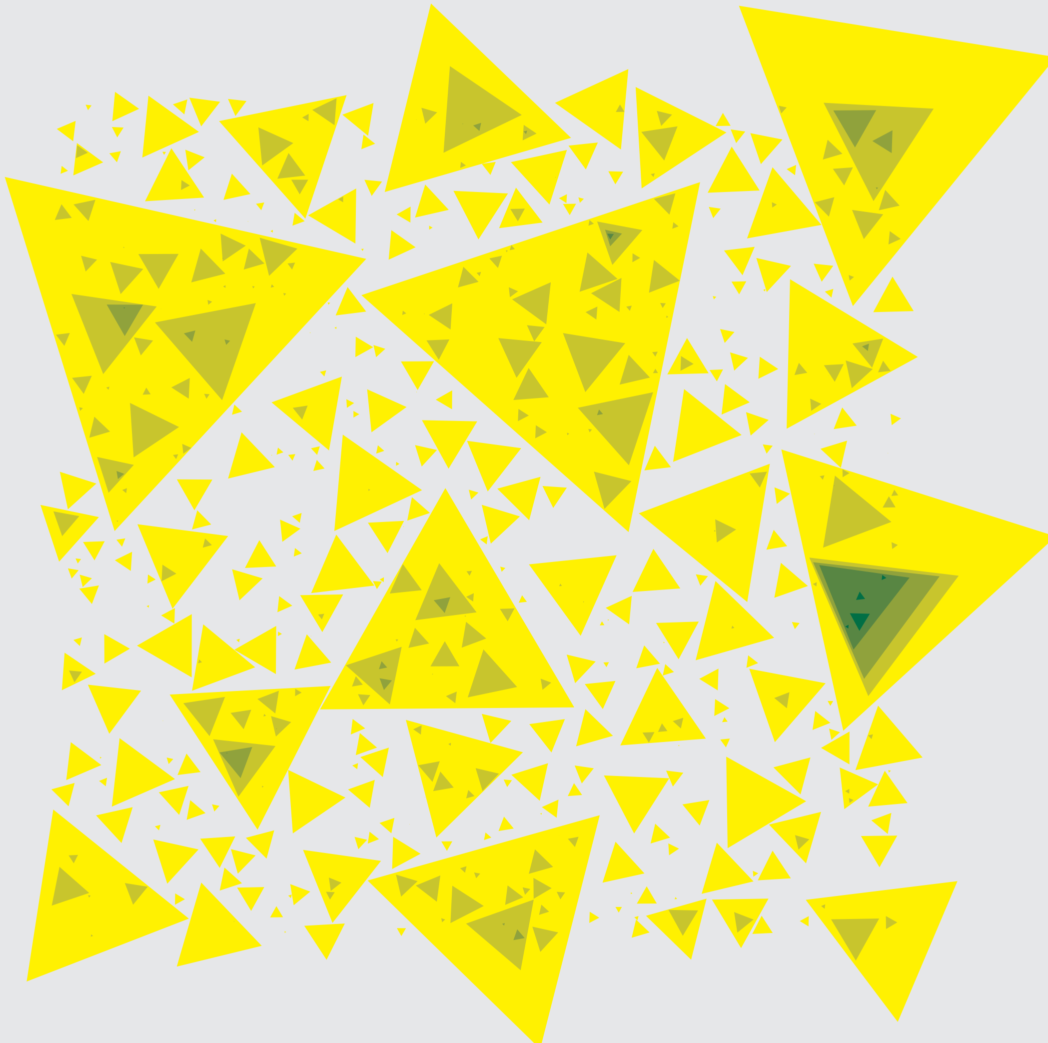

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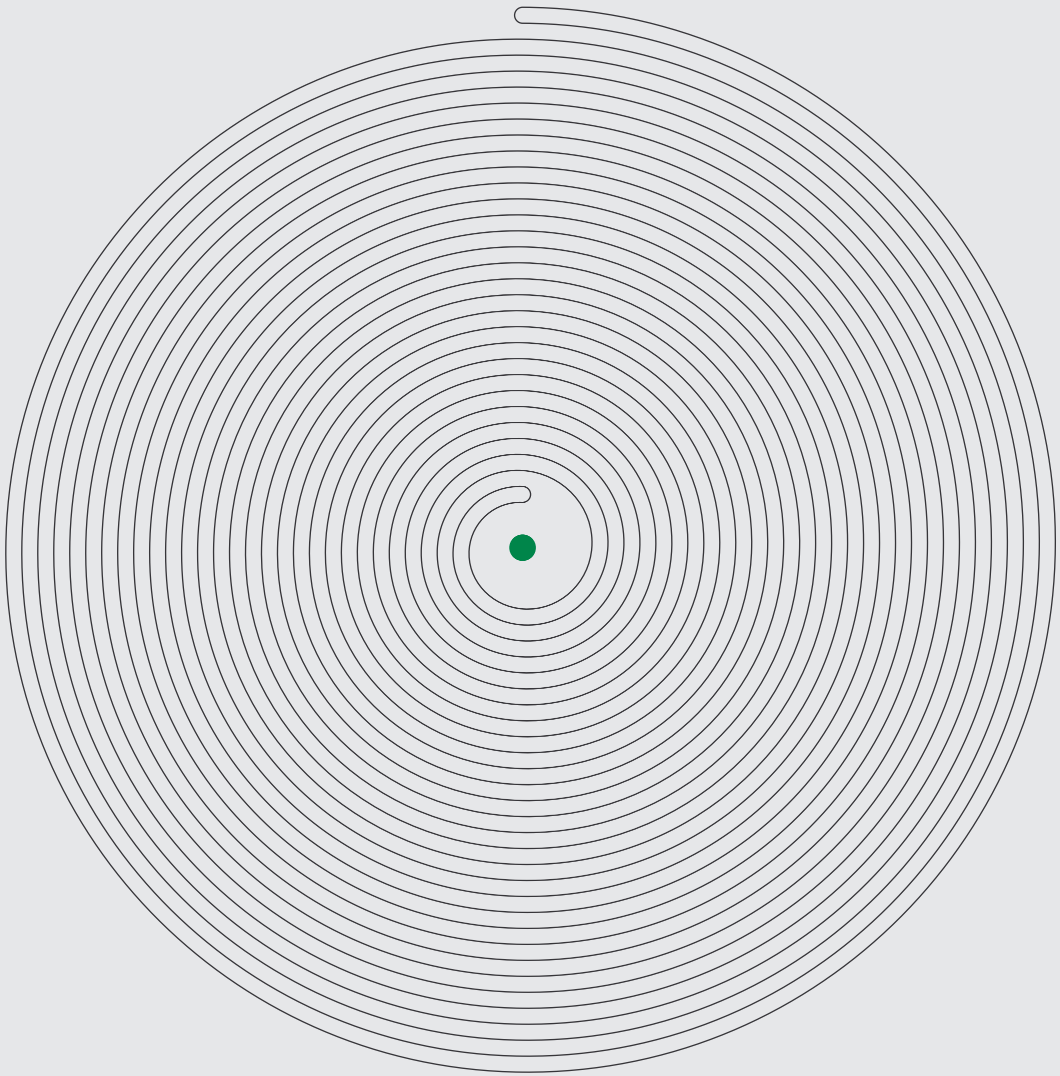

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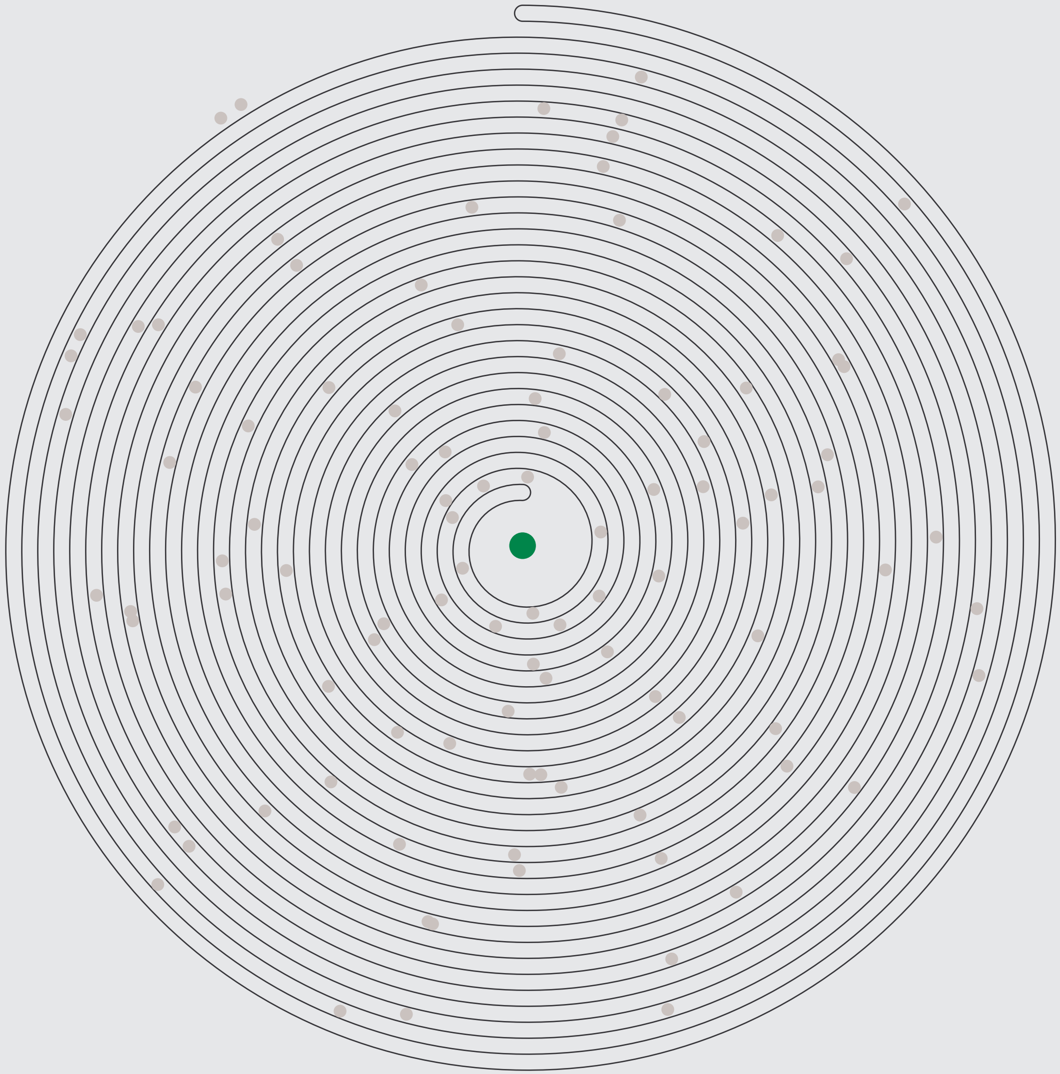

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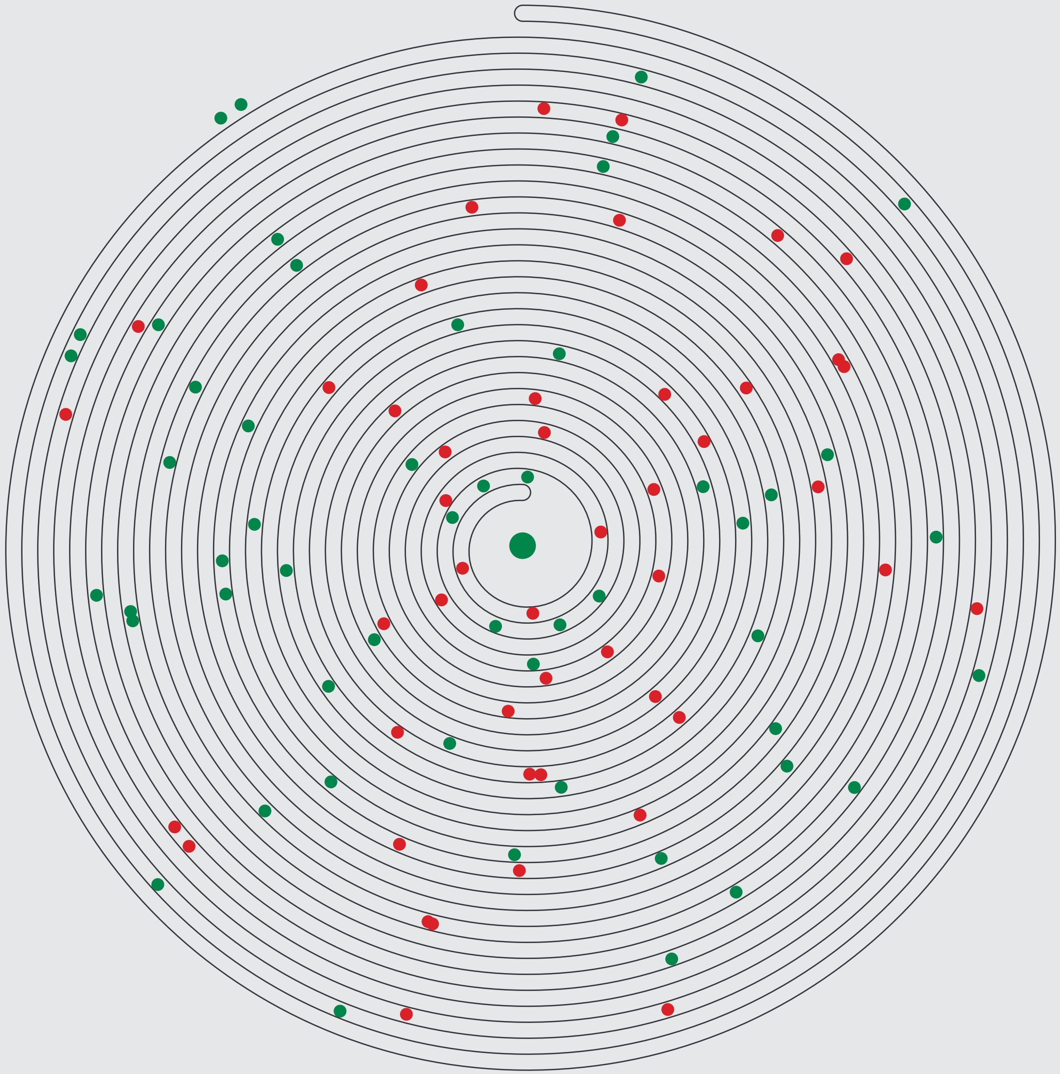

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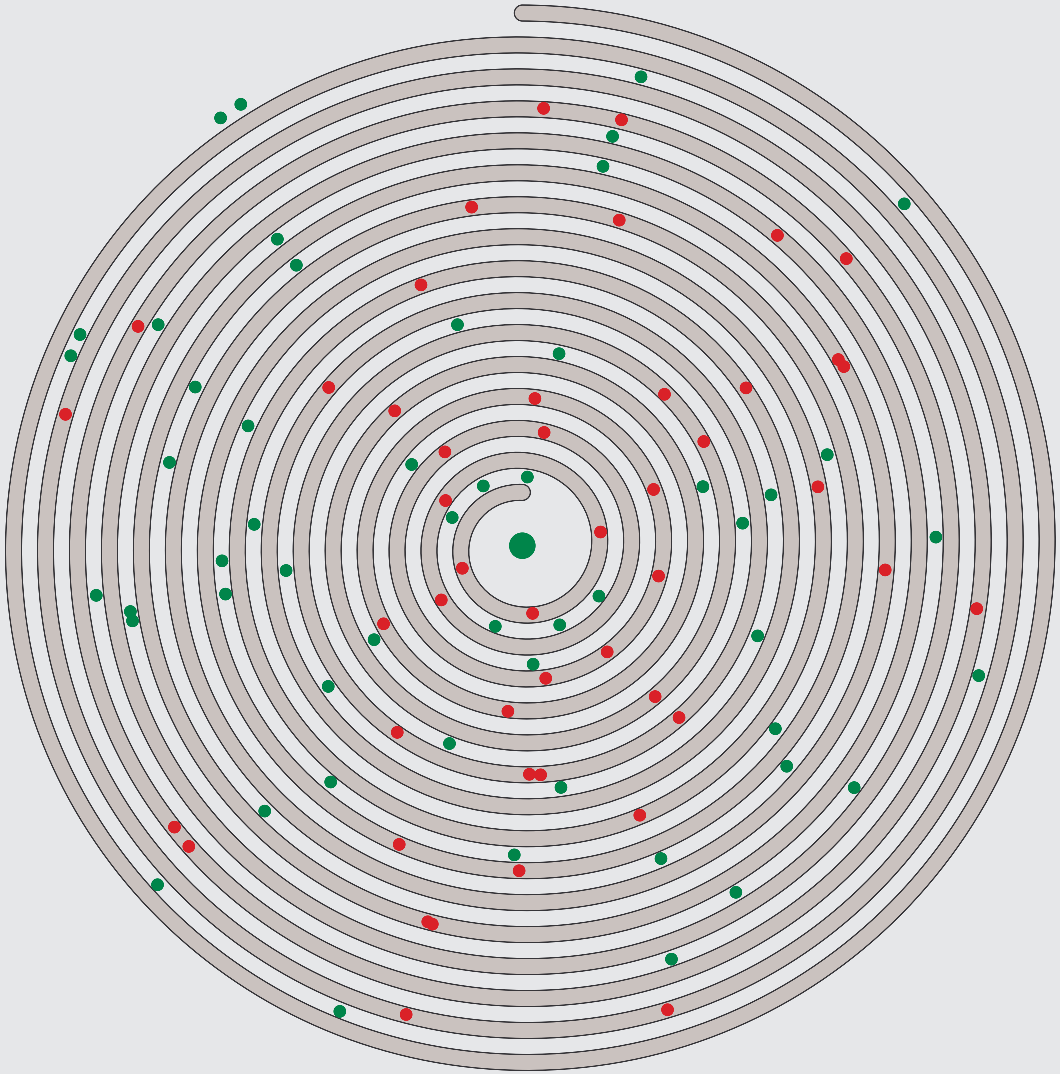
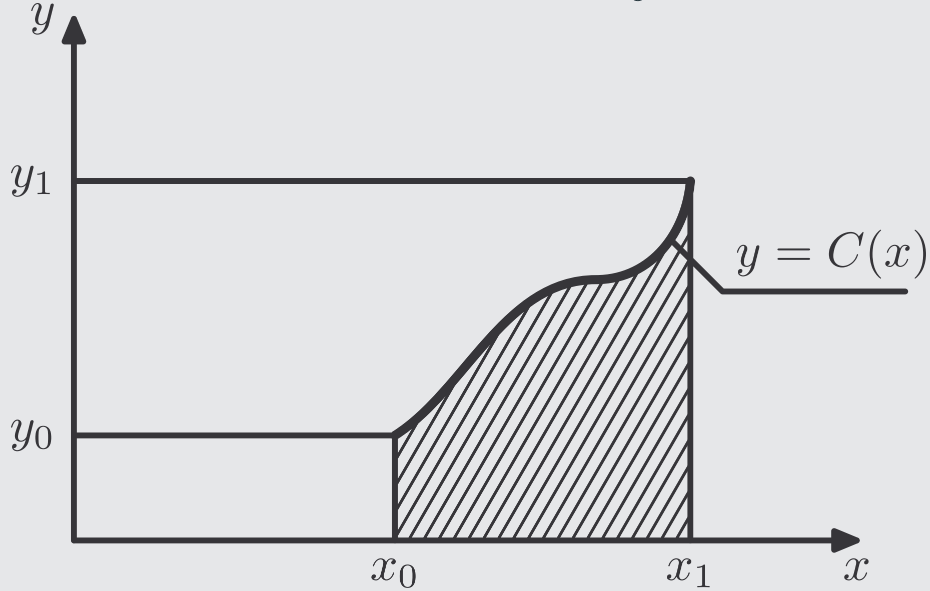


Calculating of an area

Calculating of an area

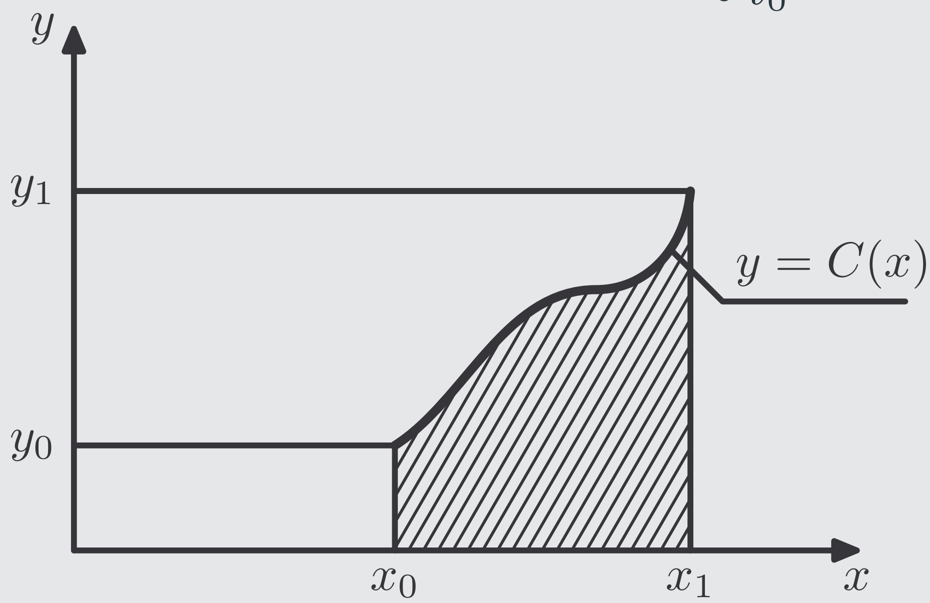
$$\text{hatched area} = \int_{x_0}^{x_1} C(x)\,dx$$



Calculating of an area

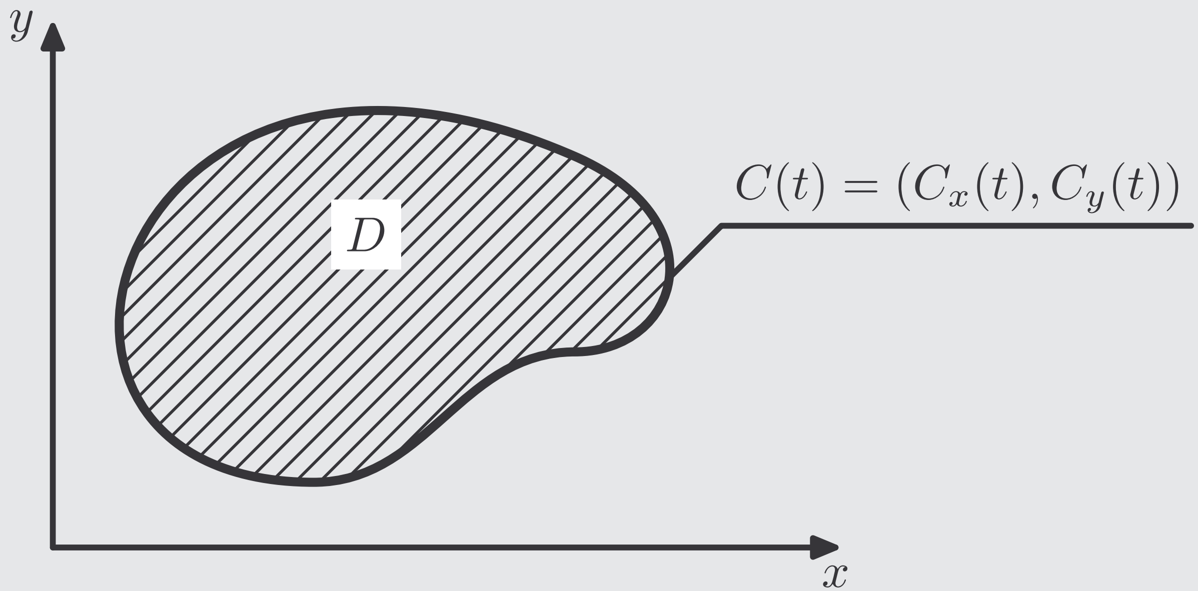
for a curve given parametrically, $x = C_x(t)$, $y = C_y(t)$,

the integral can be rewriten as $\int_{t_0}^{t_1} C_y(t)\, C_x'(t)\, dt$



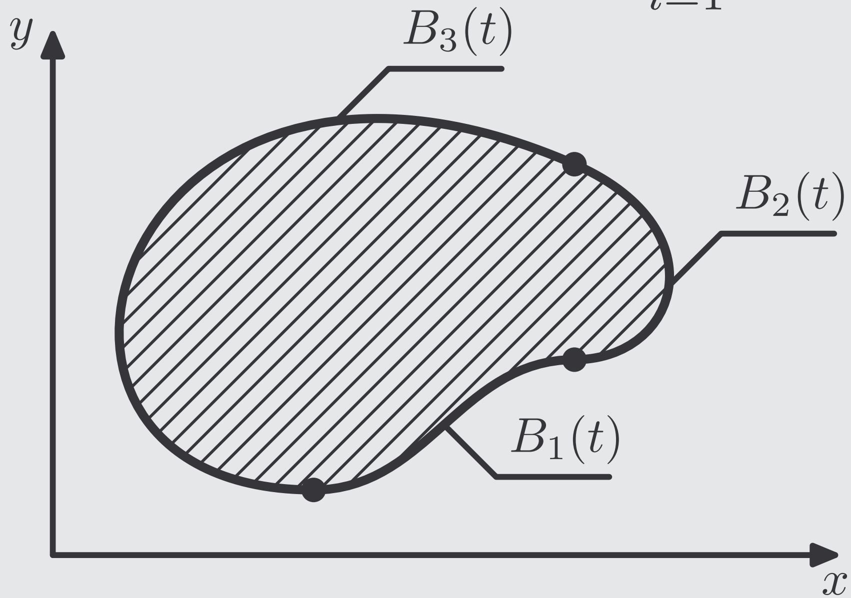
Calculating of an area

if the curve is cyclic, i.e., $C(t_0)=C(t_1)$, $t_0 \neq t_1$, the integral $D=\int_{t_0}^{t_1} C_y(t)\, C'_x(t)\, dt$ yields the area surrounded by the curve



Calculating of an area

for a contour being a Bézier spline, the area is
the sum of the relevant integrals $\sum_{i=1}^{n} \int_0^1 B_{y,i}(t)\, B'_{x,i}(t)\, dt$



Calculating of an area

The integrand expression, $B_y(t)\,B'_x(t)$, is a product of two polynomials (of the third and second degree, respectively):

$$B_x(t) = a_x(1-t)^3 + 3b_x(1-t)^2\,t + 3c_x(1-t)\,t^2 + d_x t^3,$$

$$B_y(t) = a_y(1-t)^3 + 3b_y(1-t)^2\,t + 3c_y(1-t)\,t^2 + d_y t^3,$$

therefore, finding its integral is an elementary task.

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therefore, finding its integral is an elementary task. It is the elegant, compact final formula, using only three real multiplications, that is not obvious.

$$\begin{aligned} 20\int_0^1 B_y(t)\,B'_x(t)\,dt = {} & (b_x - a_x)\,(10a_y + 6b_y + 3c_y + d_y) + \\ & (c_x - b_x)\,(4a_y + 6b_y + 6c_y + 4d_y) + \\ & (d_x - c_x)\,(a_y + 3b_y + 6c_y + 10d_y) \end{aligned}$$

Again, a built-in function computing the area seems suitable. And easily doable.


THANK YOU FOR YOUR ATTENTION